



GRAPH THEORY APPLICATIONS IN AI FOR SOCIAL NETWORK ANALYSIS

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ABSTRACT

Social Network Analysis (SNA) is a powerful interdisciplinary field that explores the patterns and dynamics of relationships between individuals, groups, organizations, and even entire societies. This article provides an overview of SNA, tracing its roots in graph theory and highlighting its various applications in fields such as sociology, computer science, business, and epidemiology. By examining the theoretical foundations of SNA and its practical implementations, this article aims to demonstrate the importance of SNA in understanding social structures, information diffusion, impact dynamics, and collective behavior. In addition, the article discusses the methodologies and tools used in SNA research, including data collection, network visualization and network metrics. Through a comprehensive analysis of SNA techniques and their applications, this article contributes to the growing knowledge in the analysis of social networks and encourages further exploration of this rich field.

INTRODUCTION

In the digital age, social media has become a key part of our daily lives, changing the way we communicate, receive information, establish social relationships, and engage in a variety of activities. Platforms such as Facebook, Twitter, Instagram, YouTube or LinkedIn allow users to create and share content, communicate with other users, actively participate in discussions on various topics, and interact with brands, companies and celebrities. It's a place where events and trends spread rapidly and information reaches millions of people in seconds. However, the enormous scale and complexity of these platforms means that the analysis of this digital ecosystem requires modern tools and methods. The formalization of the SNA began in the 1950s and 1960s with the work of researchers such as Alex Bavelas and Anatol Rapoport. They introduced mathematical graph theory to model social structures and interactions. Graph theory provided a rigorous framework for representing social networks through nodes and edges, laying the groundwork for a more systematic and quantitative approach to the study of social relationships. Social network analysis (SNA) is a methodological approach that examines patterns of social relationships and interactions between individuals, groups, organizations, or other social actors. It involves analyzing the structure of social networks, identifying key actors or nodes in the network, and understanding the flow of information, resources or influences between these nodes. In a few words, the analysis of social networks (SNA) can be described as "the study of interpersonal relationships using graph theory" (Tsvetovat & Kouznetsov, 2011). At its core, SNA is based on the understanding that social structures and relationships play a fundamental role in shaping individual behaviors, attitudes, and outcomes. By representing social interactions as a network of nodes (representing individuals or entities) connected by edges (representing relationships), SNA provides a visual and quantitative framework for the study and analysis of these social structures.

The importance of social network analysis lies in its ability to reveal hidden patterns, dynamics, and emerging properties that are not readily apparent at the individual level. It helps to discover how relationships and social connections affect the dissemination of information, the spread of behaviors or ideas, community formation, the flow of resources, and the exercise of power and influence. SNA can provide insight into social phenomena that cannot be adequately understood by focusing solely on individual attributes or characteristics. SNA has a variety of applications in

various fields. In sociology, it has been used to study social capital, social support, social impact, and social cohesion. In computer science, SNA has contributed to the development of recommendation systems, understanding of online social networks, and analyzing the diffusion of information on the web. In business, SNA has been used to explore organizational structures, collaborative networks, and diffusion of innovation. In epidemiology, SNA has proven valuable in understanding the spread of disease, identifying influential individuals in the transmission process, and designing effective intervention strategies. Overall, social network analysis provides a powerful framework for understanding the complex relationship between individuals and their social environments. It enables researchers, policymakers, and practitioners to gain insight into the structure, dynamics, and consequences of social relationships, which ultimately leads to a deeper understanding of social systems and the potential for informed decision-making and intervention. In this paper, all graphs and most calculations will be performed in Jupyter Notebook using the Python programming language.

Theoretical basis of social network analysis

The theoretical foundations of social network analysis (SNA) are rooted in graph theory, a mathematical discipline that deals with networks of interconnected nodes and edges. SNA adopts key concepts and measures from graph theory to study the complex patterns of social relationships and interactions that underlie human society. In the SNA sphere, nodes represent people, entities or social actors, while edges symbolize connections or ties between them. By mapping social interactions to the network, SNA provides a powerful visual and quantitative framework for studying the basic structure of social systems. Central to SNA are the various network metrics that shed light on the importance of nodes in the network. The centrality of the degree measures the number of node connections, which means its importance and impact in the social tissue. Interinstitutional centrality defines the extent to which a node serves as a bridge between other nodes, acting as a critical link for the flow of information or impact. Proximity centrality, on the other hand, evaluates the average distance between a node and all other nodes, emphasizing its availability on the network.

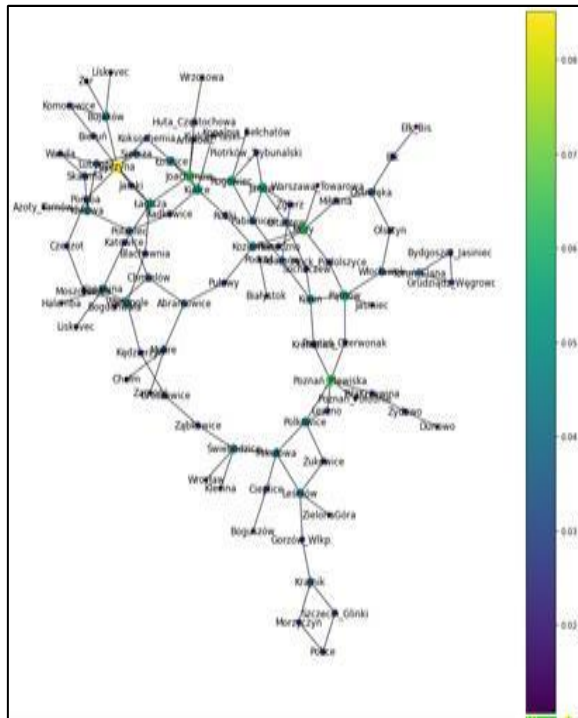


Figure 1. 220kV network graph with degree centrality.

SNA delves into network properties such as the clustering factor, which reveals the tendency of nodes to form groups or clusters, suggesting the presence of coherent sub-structures. The phenomenon of the small world emphasizes the efficiency of social networks, since most nodes can be reached from any other node with an extremely small number of steps.

With SNA, researchers gain valuable insight into the dynamics of social systems. By studying the flow of information, influence, and behavior through social networks, SNA allows us to capture the diffusion of innovation, the spread of rumors, and the mechanisms behind collective decision-making.

The theoretical foundations of SNA go beyond traditional sociology to include computer science, business, epidemiology, political science, and more. He revolutionized fields such as computer science, contributing to the development of recommendation systems, link prediction algorithms and understanding of online social networks. The theoretical basis constantly shapes the evolution of the SNA. Researchers are exploring advanced methodologies for analyzing multi-layer and multiplex networks, integrating different types of relationships to gain a comprehensive understanding of social interactions. Moreover, the application of machine learning, natural language processing, and advanced statistical techniques enhances SNA's analytical prowess, opening up new dimensions of

social network analysis.

In summary, the theoretical foundations of social network analysis, rooted in graph theory, provide a scaffolding for understanding the complex web of social connections that shape human behavior, communication, and influence. As SNA progresses, researchers must address challenges such as ethical considerations, data quality, and scalability while exploring new frontiers of interdisciplinary applications. With its enduring importance, SNA can make a significant contribution to our understanding of social systems and informing meaningful decisions for a more connected world.

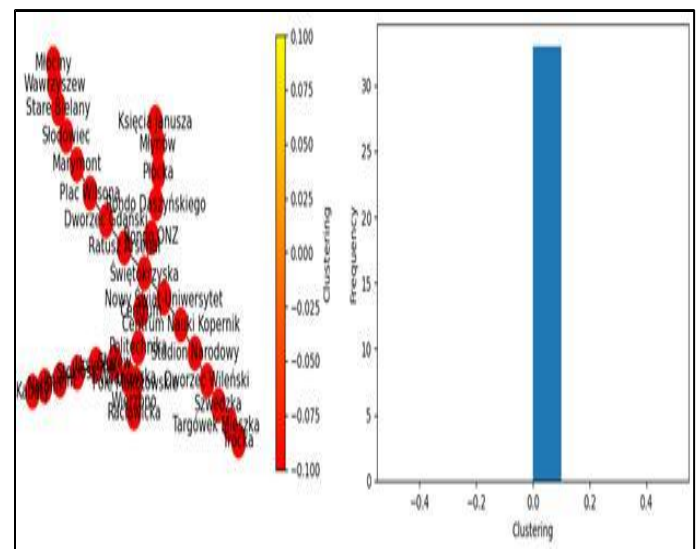


Figure 2. Grouping factor (Warsaw metro network).

Methodologies in the analysis of social networks

Social Network Analysis (SNA) methodologies include a diverse array of techniques that enable researchers to study, analyze, and interpret the complex structure of social networks. These methodologies play a key role in collecting network data, representing relationships, visualizing structures, and extracting valuable information from the inter-connectedness of social actors.

One of the fundamental aspects of the SNA methodology is data collection. Researchers use a variety of techniques, such as surveys, observations, and the use of on-line data sources, to gather information about social ties and interactions. Surveys allow individuals to provide valuable data on their relationships, while observational methods offer a more direct approach to studying social interactions in real-world settings. In addition, the growing prevalence of online social media platforms has opened up new opportunities for data

collection, allowing researchers to delve into online social networks and study virtual communities.

The representation of network data is another critical aspect. SNA uses two basic methods: the neighborhood matrix and the neighborhood list. The neighborhood matrix represents nodes and their relationships as a square matrix, providing a clear overview of the network structure. A neighborhood list, on the other hand, represents nodes as a list, along with their connected nodes, which is more efficient for infrequent networks with fewer connections.

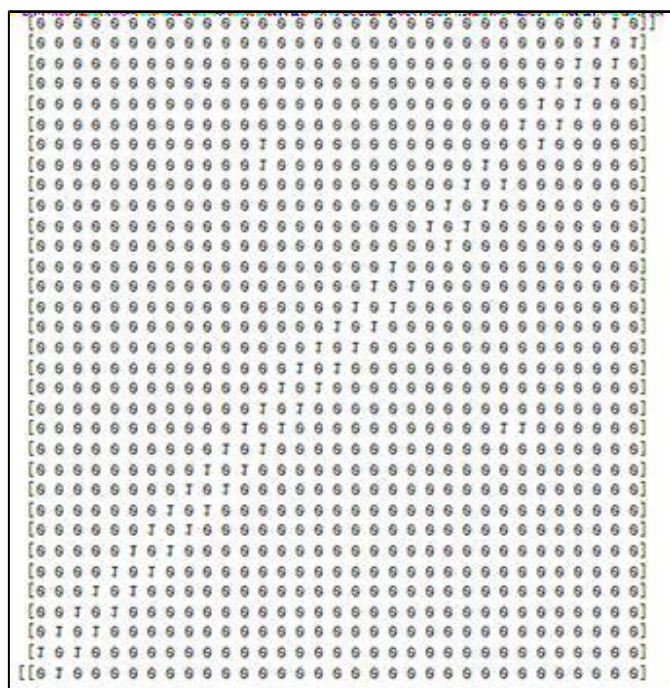
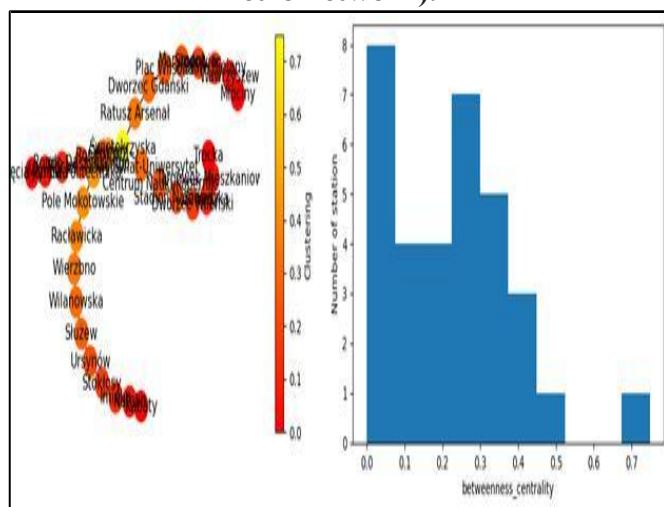


Figure 3. Neighbourhood matrix (Warsaw metro network).



lines connecting them. This visual image makes it easy to identify key actors, communities, and structural patterns in the network. Matrix diagrams are another visualization method in which the network is represented in the form of a matrix, allowing a clear view of relationships and connections.

Sampling methods are used to manage the computational complexity of large networks and provide generalization. Random sampling involves randomly selecting a subset of nodes or edges, while snowball sampling starts with a few known nodes and expands to collect additional nodes nominated by existing ones. Properly addressing missing data is essential, as incomplete or unavailable data can lead to biased analysis and interpretation. Thanks to proper planning, we can better use human work, as well as machines used in transport, production or distribution of goods and services (Alsina, 2011).

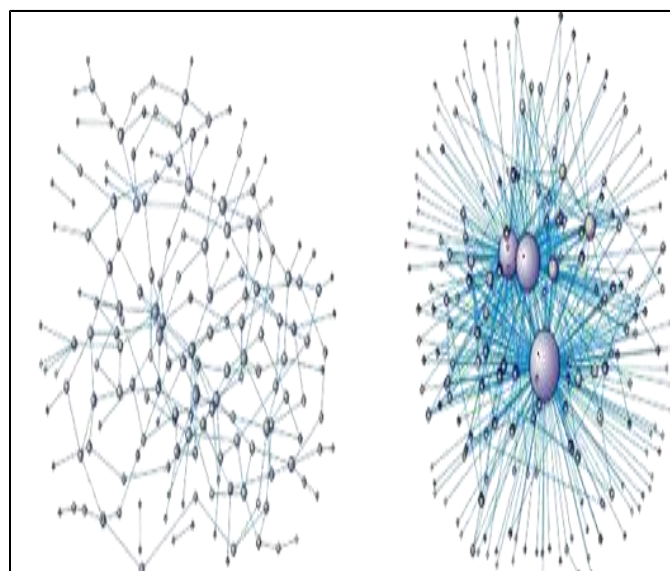


Figure 4. Distribution of vertex degrees.

Visualization techniques are essential for understanding the complex structures of social networks. Node connection diagrams offer a visually appealing representation in which nodes are represented as points or circles and edges as

Analyzing and interpreting social networks is based on a wide range of network indicators and algorithms. Measures of centrality, such as degree, interinstitutionality, and centrality of proximity, help identify influential actors, bridges, and people who are well connected in the network. Community detection algorithms reveal consistent sub-groups, providing insight into group dynamics and social grouping. In addition, studying the dynamics of networks over time is critical to understanding how relationships evolve and adapt to changing circumstances.

Figure 5. Interinstitutional centrality (Warsaw metro network)

Looking ahead, the future of SNA methodology includes exploring multi-layered and multiplex networks, integrating contextual information, and combining SNA with other analytical approaches such as machine learning and natural language processing. As methodologies advance, SNAs will continue to unlock new dimensions of social network analysis, enabling researchers to gain deeper insight into the complex dynamics of human interaction and their far-reaching implications across fields.

Applications of social network analytics

Social Network Analytics (SNA) offers a wide range of applications in different fields, each of which harnesses the power of network relationships to uncover valuable insights and inform decision-making processes. In the field of sociology, SNA plays a key role in understanding social structures and dynamics. Researchers use SNA to identify communities on social networks, revealing subcultures and interaction patterns. In addition, SNA helps study the diffusion of information, behavior, and opinion in social systems, shedding light on social impact and the spread of innovation. Computer science finds great value in SNA for building personalized recommendation systems. By analyzing social connections and user preferences, SNA increases the accuracy of recommendations across e-commerce platforms, social media, and content distribution networks. In addition, link prediction algorithms, derived from SNA principles, predict future connections, enabling better friend recommendations, targeted marketing strategies, and efficient collaborative networks.

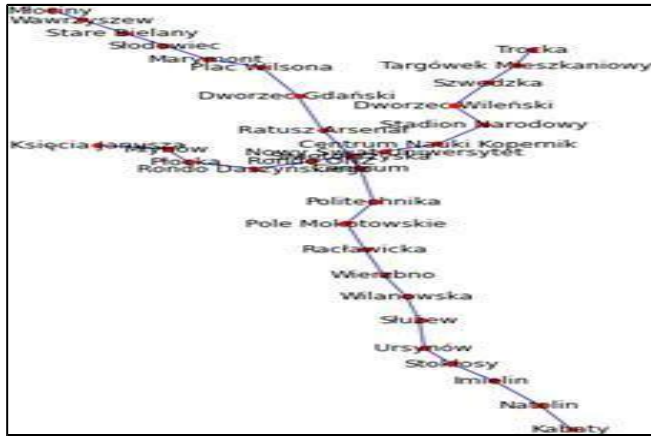
The business world benefits from SNA by gaining insight into organizational structures and collaboration patterns. By exploring communication networks in companies, SNA uncovers the formal and informal dynamics of power, facilitating effective decision-making and knowledge transfer. Social network analysis (SNA) has become an invaluable tool for companies and organizations, providing valuable information about their internal dynamics and social interactions. By visualizing and analyzing communication patterns and relationships within a company, SNA helps uncover hidden structures and optimize collaboration. One of the key applications of SNA in companies is to understand organizational

structure and communication flows. By mapping the connections between employees and teams, managers can gain comprehensive insight into the formal and informal relationships that influence decision-making and the dissemination of information. This knowledge allows for more effective coordination, identification of communication bottlenecks and filling gaps between departments. SNA allows you to indicate techniques for encouraging the individual to express knowledge and transmitting it in the organization and giving it formal rules understandable to employees (Adamus-Matuszyńska 2013). What is more, SNA helps businesses leverage social media marketing by identifying influencer users and detecting trending topics for successful brand promotion. In the field of social media, SNA has changed the way platforms engage their users. Using SNA techniques, social media platforms gain insight into user behavior, content consumption patterns, and influencer users. Using this knowledge, they can create personalized experiences for users by recommending content that aligns with their interests and preferences.

In the field of epidemiology and public health, SNA is becoming a key tool in understanding the spread of infectious diseases. Governments, NGOs, experts and epidemiologists are trying to use models to understand how to respond, fight and treat a pandemic (Alguliyev et al. 2021). By mapping social contacts, researchers identify high-risk individuals, develop targeted vaccination strategies, and conduct contact tracing during disease outbreaks. Political science and policy analysis harnesses the potential of SNA to study policy networks and implement policy. By analyzing the relationships between policymakers, interest groups, and stakeholders, SNA uncovers hidden power structures and the flow of political influence, helping to effectively evaluate policy and make decisions. The analysis of social networks has proven to be a valuable tool in various real-world scenarios. For example, during the COVID-19 pandemic, SNA played a key role in controlling and containing the disease. Public health officials have used SNA to track and understand the spread of the virus by analyzing social interactions and identifying high-risk individuals. Through contact tracing efforts, SNA has helped map networks of infected individuals and their contacts, enabling targeted interventions and isolating potential transmission chains. This approach has proven crucial in limiting the spread of the virus and guiding public health strategies.

In addition, SNA applications cover

interdisciplinary fields. In urban planning and transportation, SNA helps optimize traffic flow and identify transportation hubs based on connectivity in the city. In finance, SNA offers insight into the resilience of financial systems and the impact of contagion during economic crises. In addition, SNA helps energy companies understand energy distribution networks, optimize supply chains, and



identify key players in energy consumption patterns. As SNA continues to evolve, emerging trends promise exciting opportunities. The analysis of online social networks and user behavior remains a living area of exploration, shedding light on the complexity of the virtual landscape. The integration of SNA with machine learning and natural language processing has the potential for powerful predictive models and semantic network analysis. Moreover, the use of SNA to design and evaluate network interventions, promote positive behavior change and mitigate negative effects represents a promising way forward.

Figure 6. Warsaw metro network

Academic researchers also use SNA to gain insight into scientific collaboration. By examining co-authorship patterns and citation networks, SNA reveals influential researchers and the flow of knowledge in scientific communities. This information helps to understand emerging research trends, interdisciplinary collaboration and disseminate breakthroughs. In addition, SNA helps you understand online communities and foster positive interactions. By detecting distinct groups or communities on social networks, platforms can implement effective content moderation and create environments that encourage constructive discussions. This helps maintain platform integrity and user satisfaction while reducing the risk of misinformation and toxic behavior.

Structural holes are gaps or missing connections between people in a network. The individuals who fill these gaps are known as “brokers.” SNA helps to identify structural holes and brokers in social networks. Intermediaries play a key role in facilitating the flow of information and access to different resources in different parts of the network. In sociology, understanding structural holes is essential to analyze the dynamics of power and social capital. Intermediaries often have a unique advantage in controlling the flow of information and have access to more diverse perspectives, which can increase their influence and opportunities for social and economic benefits. For example, in an organizational context, identifying intermediaries that bring together different departments can lead to better knowledge sharing and increased efficiency. In economic networks, brokers can use their position to broker transactions and create value by bridging gaps between separate entities. Social network analysis is a powerful tool in sociology, allowing researchers to explore community structures, social impact dynamics, and the importance of structural holes in shaping social interactions and behaviors. By discovering these patterns, SNA contributes to a deeper understanding of human social systems and enables more effective interventions and strategies in a variety of sociological contexts. In criminal investigations, social network analysis (SNA) has become a powerful tool for understanding and dismantling criminal networks. Law enforcement and investigators use SNA to analyze the complex web of relationships between criminals and their accomplices. Representing criminal activity and relationships as a network of nodes and edges, SNA allows investigators to visualize the structure of criminal networks and identify key actors within them. Nodes represent persons or entities involved in criminal activity, while edges represent connections or relationships between them, such as joint criminal ventures, communications or financial transactions. One of the main goals of the SNA in criminal investigations is to identify and target influential nodes, often referred to as “kings” or “crime bosses.” These are people who occupy central positions in the network and play a key role in coordinating criminal activities, distributing resources and controlling their criminal enterprise. With the SEN, investigators can trace the flow of information, contraband or illicit funds within a criminal network. This knowledge helps to understand the modus operandi of criminal operations, identify patterns of criminal behavior and predict potential future criminal activities.

Moreover, SNA can uncover hidden links between seemingly unrelated criminal actors. It helps to expose the existence of bridges or “brokers” who act as intermediaries, facilitating illegal exchanges or enabling cooperation between different criminal groups. The identification of intermediaries is essential for dismantling criminal networks, as they can be targeted to break links between criminal actors that would otherwise be disconnected. In addition to identifying key actors and their relationships, SNA helps gather key evidence for litigation. By analyzing communication patterns and identifying common criminal connections, an SNA can provide supporting evidence to support other investigative activities. SNA also contributes to strategic intelligence in criminal investigations. By exploring the overall structure and patterns of criminal networks, law enforcement agencies gain a deeper understanding of the broader criminal ecosystem, helping to develop more effective long-term strategies to prevent and disrupt crime. However, ethical issues and data privacy must be taken into account when conducting SNA in criminal investigations. Ensuring the protection of individuals’ rights and data integrity is crucial to maintaining the credibility and validity of SNA findings in court.

In summary, the applications of social network analysis span a broad spectrum of fields, providing valuable insight into the complex web of social relationships and their profound impact on individuals, communities, and society as a whole. The versatility of SNA, combined with continuous progress, ensures that it continues to be an indispensable tool in exploring the complex dynamics of human interaction and driving positive change in various fields.

Example of using network analysis

This research seeks to conduct an in-depth analysis and comparison of sample social networks using graph theory. The primary objective extends beyond the mere calculation of network parameters. Rather, it aims to uncover the underlying dynamics and nuances within the selected networks. The study is designed to be comprehensive, considering various aspects of network topology and statistical characteristics.

In the initial phase, a diverse set of social networks will be curated, ensuring representation across different sizes, structures, and purposes. Importantly, only networks with a minimum of two connections will be included to facilitate meaningful analysis. The analysis will encompass

a thorough investigation of graph theory parameters, including node degree, centrality measures, and connectivity patterns. The goal is not only to identify these parameters but to explain observed differences across the diverse set of social networks. Moving beyond individual parameters, the study will scrutinize topological characteristics such as average node degree, Pearson correlation coefficient, network efficiency, and clustering coefficient. These metrics aim to reveal the intricate relationships and patterns within the networks, providing insights into their organizational principles and information flow dynamics. Additionally, the research will explore statistical characteristics, delving into node degree distribution and shortest path distribution. This analysis is crucial for understanding the variability and robustness of the networks under examination. Given the potential presence of disconnected components in some networks, a pragmatic approach will be adopted. The “connected_components” method will be employed to calculate parameters for interconnected components, ensuring a comprehensive understanding of network structures.

The findings from both graph theory and topological analyses will be integrated to derive overarching conclusions about the characteristics and behaviors of the sample social networks. Practical implications of the research will be discussed, considering how observed network characteristics may impact real-world scenarios in fields such as social sciences, communication studies, or network optimization.

Furthermore, the research will suggest future directions, proposing additional parameters for exploration, considering the impact of external factors on network dynamics, and advocating for the application of advanced graph theory concepts. Through this expanded goal, the study aims to provide a nuanced and insightful perspective on the intricate dynamics of social networks, contributing to both theoretical understanding and practical applications. The test results will be shown in the table “Topological characteristics of social networks”.

	N	M	< k >	Max k	r	d	E	< l >	C
Network no.1	22	26	2,363	6	- 0,221	8	0,337	3,342	0,179
Network no.2	34	41	2,412	8	- 0,177	9	0,340	3,780	0,034
Network no.3	11	10	1,818	3	0,184	3	0,326	2,357	0,212
Network no.4	15	15	2,0	4	- 0,602	7	0,415	3,238	0,0
Network no.5	24	26	2,167	6	- 0,323	9	0,347	3,238	0,051
Network no.6	17	18	2,4	4	- 0,204	10	0,372	4,029	0,166
Network no.7	13	14	2,154	5	- 0,538	5	0,475	2,718	0,077
Network no.8	17	17	2,0	4	0,177	7	0,330	3,2	0,225
Network no.9	15	13	1,733	4	- 0,099	6	0,460	2,987	0,128
Network no.10	15	13	1,733	3	- 0,262	8	0,286	3,485	0,0
Network no.11	2	1	1	1	-----	1	1	1	0,0

Table 1. Topological characteristics of social networks

N number of vertices, M number of edges, < k > average vertex degree, Max k maximum vertex degree, r Pearson degree correlation, d network diameter, E network efficiency, < l > average shortest path, C clustering coefficient.

In conclusion, the examination of the topological characteristics of the analyzed social networks reveals a rich tapestry of structural diversity and connectivity dynamics. The variations in network sizes, ranging from a modest 2 vertices to a more intricate 34 vertices, underscore the heterogeneous nature of these social structures. The number of edges, indicative of interconnections, further emphasizes the complexity inherent in these networks.

The average vertex degree (< k >) showcases the diverse interconnectivity patterns among nodes, while the maximum vertex degree (Max k) highlights the presence of influential hubs with substantial connections. From the analysis we can conclude that < k > does not depend on the size of the network. Properties such as the average vertex degree or the maximum node degree can be used for further analysis, including examining the empirical distribution of the node degree. The scope of the nodal degree is similar to other networks. The Pearson degree correlation (r)

provides valuable insights into the assortativity or disassortativity of the networks, elucidating the nature of relationships between nodes.

Network diameter (d) and efficiency (E) elucidate the varying degrees of accessibility and information flow within the networks. Taking into account the diameter and efficiency of the network, in theory the first value (diameter) should increase as the network size increases, and the second value (efficiency) should decrease with the increase. Although in the first case the analysis showed that this thesis was correct, it did not necessarily prove true in terms of efficiency. The size of the network (number of vertices and nodes) has no impact on performance. This is because although the network has more connections, the distance between the vertices is smaller and the network is more condensed than networks with fewer vertices, which can be seen in the appropriate graphs. The range in average shortest path length (< l >) underscores differences in the speed of information dissemination across these social structures. The clustering coefficient (C) reflects the propensity of nodes to form local clusters, influencing the overall network cohesion. The results of the clustering coefficient can be compared with random graphs with the same number of vertices and edges, using the fact that the coefficient in such a graph is equal to the probability of randomly selecting an edge from all possible edges.

It is noteworthy that smaller networks, as observed in Network no. 11, introduce unique challenges in drawing statistically robust conclusions. Caution is warranted in generalizing findings from such limited datasets.

Overall, these insights into the topological nuances of social networks contribute to a deeper understanding of their functionality and organizational principles. The diverse array of parameters examined provides a comprehensive view, paving the way for future research avenues that delve into the specific contexts and applications of these social structures. This study lays the groundwork for further exploration into the intricate dynamics that govern social networks and their implications in real-world scenarios.

Challenges and future directions

Social Network Analysis (SNA) faces various challenges that researchers and practitioners must face in order to fully exploit its potential. Ethical considerations are important because SNA often deals with personal and sensitive data. Striking a balance between data availability and privacy protection is key to ensuring responsible use of data.

Acquiring high-quality data and eliminating bias are ongoing challenges at SNA. Incomplete or biased data can skew analyses and lead to misleading conclusions. Researchers must carefully select and pre-process data to ensure their reliability and validity. SNA scalability presents a computational challenge as networks become larger and more complex. Efficient algorithms and visualization techniques are required to handle the vast amounts of data generated by vast social networks. Understanding dynamic networks that change over time requires novel methodologies. Analyzing changing relationships and capturing time patterns requires a sophisticated approach to accounting for time-varying data. Multi-layer and multiplex networks create new complexities. Social interactions often occur in different contexts, requiring the integration and analysis of multiple layers of connections. Researchers need innovative techniques to study the interaction between different network dimensions.

Network sampling and representativeness are essential to draw meaningful conclusions. Researchers need to make sure that the samples they choose accurately represent the entire network, avoiding bias and generalizing the results more effectively. Handling missing data is a constant challenge. The development of robust imputation methods is crucial to address missing information without introducing bias.

Integrating SNA with other analytical techniques, such as machine learning, creates both opportunities and challenges. Combining methodologies can improve predictive modeling and unlock deeper insights, but it also requires interdisciplinary collaboration and methodological consistency. Addressing these challenges, the future of SNA remains promising. Advanced methodologies will continue to emerge, providing innovative solutions for scalability, dynamic networks, and multi-tier structures. The use of large data sets and computational advances will allow the analysis of even larger and more complex social networks. Contextual information will enrich SNA analytics by offering a deeper understanding of social interactions in specific settings. Researchers can study the influence of geographical, temporal and semantic factors on network structures and dynamics.

The potential of network interventions is an exciting avenue for SNA. Designing and evaluating interventions can drive positive behavior change, optimize social systems, and solve pressing social problems. The interdisciplinary applications of SNA will expand, fostering collaboration with

fields such as neuroscience, economics and climate science. These interdisciplinary approaches will provide novel insights and solve complex problems from multiple points of view.

Explainable AI in SNA will improve the interpretability of the results, making the findings more accessible and feasible for policymakers and stakeholders. By prioritizing ethical data practices, researchers will respect transparency, privacy, and responsible use of data in SNA research.

In conclusion, SNA challenges offer opportunities for growth and innovation. Taking future directions, researchers can gain deeper insight into the complexity of social networks and their impact on individuals and society. With ethical considerations in mind, SNA will continue to empower informed decision-making, drive positive change, and provide a clearer understanding of human interaction and social systems.

Summary

Social Network Analysis (SNA) offers a powerful framework for understanding the structure, properties, and dynamics of social networks. Using concepts from graph theory and using various methodologies, SNA enables scientists to discover hidden patterns, study the diffusion of information, analyze the dynamics of influence and study collective behavior in social networks, as well as the dependencies of structure on the attributes of individuals and the impact on the processes that occur through relationships (transactions, information flow, and cooperation) (Kawa, 2013).

The importance of SNA lies in its ability to provide insights that cannot be obtained by focusing solely on individual attributes or characteristics. It allows us to understand how social relationships shape behaviors, attitudes, and outcomes at both the micro and macro levels. SNA finds applications in a variety of fields, including sociology, computer science, business, epidemiology, political science, and more. Although SNA has made significant contributions, challenges remain. Ethical considerations, data quality and availability, scalability and methodological progress are areas that require attention. Overcoming these challenges will pave the way for future progress at SNA. Looking ahead, SNA holds promising opportunities. Studying multi-layer and multiplex networks, integrating contextual information, and combining SNA with other analytical approaches are areas of ongoing research. In addition, the impact of online social networks, the design of network interventions and interdisciplinary applications offers exciting opportunities for exploration.

In conclusion, SNA continues to be a valuable tool for researchers, practitioners and policymakers in understanding and analyzing social phenomena. By addressing challenges, pursuing methodological advances, and discovering new applications, SNA will contribute to a further understanding of complex social systems, ultimately supporting informed decision-making and improving the well-being of individuals and communities. The overarching goal is to contribute to the understanding of social network structures, fostering insights that can inform various fields, including sociology, communication studies, and network optimization. Through the exploration of diverse parameters and methodologies, the research aims to uncover patterns, variations, and potential applications within the complex landscape of social connections.

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